1. Study the behavior of a plastic synapse described by the Markram-Tsodyks formalism. The synaptic efficacy after the $n^{th}$ spike is proportional to $u_n R_n$. The time evolution is given by:

$$u_{n+1} = u_n \exp(-\Delta t / \tau_{facil}) + U(1 - u_n \exp(-\Delta t / \tau_{facil}))$$  \hspace{1cm} (1)$$

$$R_{n+1} = R_n(1 - u_{n+1}) \exp(-\Delta t / \tau_{rec}) + 1 - \exp(-\Delta t / \tau_{rec})$$  \hspace{1cm} (2)$$

where $\Delta t$ is the time between spike $n$ and spike $n + 1$.

- Assuming that the spikes are generated periodically with a period $T = 1/f$, evaluate the synaptic efficacy as a function of $f$.
- Compare the previous result with the one obtained for a Poisson process with the same average frequency.
- Explore the space of parameters $U; \tau_{facil}; \tau_{rec}$. Consider the cases $U \ll 1, U \approx 0.5, \tau_{facil} \ll \tau_{rec}, \tau_{facil} \gg \tau_{rec}$.

2. Estimate the capacity of the Hopfield model without noise.

- Create the patterns $\xi^\mu_i (i = 1, ..., N; \mu = 1, ..., p)$. Each one of the values is $\pm 1$ with equal probability.
- Evaluate the matrix of connections: $J_{ij} = \frac{1}{N} \sum_{\mu=1}^{p} \xi^\mu_i \xi^\mu_j$ (take $J_{ii} = 0$).
- Take each one of the patterns as initial condition and iterate until the dynamic converges to a fixed point ($s^\mu_i$).
- Evaluate the overlap $m^\mu = \frac{1}{N} \sum_{i=1}^{N} s^\mu_i \xi^\mu_i$.
- Repeat for all the patterns and evaluate the distribution of overlaps.
- Repeat for $N = 500, 1000, 2000, 4000$ and $\alpha = p/N = 0.12, 0.14, 0.16, 0.18$.

3. Simulate the dynamics of the Hopfield model with noise using:

$$Pr(s_i(t + 1) = \pm 1) = \frac{\exp(\pm \beta h_i(t))}{\exp(\beta h_i(t)) + \exp(-\beta h_i(t))}$$  \hspace{1cm} (3)$$
where \( h_i(t) = \sum_{j=1}^{N} J_{ij} s_j(t) \). Take as initial condition each one of the patterns \( (\xi_i^{\mu}) \). Apply the rule and after visiting each site 10 times evaluate the overlap. Take \( N = 4000, p = 40 \) and plot the average overlap as a function of \( T = 1/\beta \), for \( T = 0.1, 0.2, \ldots, 2 \).