LASCON VIII

Tutorials

1. Study the behavior of a plastic synapse described by the Markram-Tsodyks formalism. The synaptic efficacy after the n^{th} spike is proportional to $u_n R_n$. The time evlution is given by:

$$u_{n+1} = u_n \exp(-\Delta t/\tau_{facil}) + U(1 - u_n \exp(-\Delta t/\tau_{facil})) \quad (1)$$
$$R_{n+1} = R_n(1 - u_{n+1}) \exp(-\Delta t/\tau_{rec}) + 1 - \exp(-\Delta t/\tau_{rec}) \quad (2)$$

   where Δt is the time between spike n and spike $n+1$.

   - Assuming that the spikes are generated periodically with a period $T = 1/f$, evaluate the synaptic efficacy as a function of f
   - Compare the previous result with the one obtained for a Poisson process with the same average frequency.
   - Explore the space of parameters $U, \tau_{facil}, \tau_{rec}$. Consider de cases $U \ll 1$, $U \approx 0.5$, $\tau_{facil} \ll \tau_{rec}$, $\tau_{facil} \gg \tau_{rec}$

2. Estimate the capacity of the Hopfield model without noise.

   - Create the patterns ξ_i^μ $(i = 1, ...N; \mu = 1, ..., p)$. Each one of the values is ± 1 with equal probability.
   - Evaluate the matrix of connections: $J_{ij} = \frac{1}{N} \sum_{\mu=1}^{p} \xi_i^\mu \xi_j^\mu$ (take $J_{ii} = 0$).
   - Take each one of the patterns as initial condition and iterate until the dynamic converges to a fixed point (s_i^μ).
   - Evaluate the overlap $m^\mu = \frac{1}{N} \sum_{i=1}^{N} s_i^\mu \xi_i^\mu$.
   - Repeat for all the patterns and evaluate the distribution of overlaps.
   - Repeat for $N = 500, 1000, 2000, 4000$ and $\alpha = p/N = 0.12, 0.14, 0.16, 0.18$

3. Simulate the dynamics of the Hopfield model with noise using:

$$Pr(s_i(t+1) = \pm 1) = \frac{\exp(\pm \beta h_i(t))}{\exp(\beta h_i(t)) + \exp(-\beta h_i(t))} \quad (3)$$

where $h_i(t) = \sum_{j=1}^{N} J_{ij} s_j(t)$. Take as initial condition each one of the patterns (ξ_i^μ). Apply the rule and after visiting each site 10 times evaluate the overlap. Take $N = 4000, p = 40$ and plot the average overlap as a function of $T = 1/\beta$, for $T = 0.1, 0.2, ..., 2$.

4. Analayze the bistabiluty properties of a two population network with short term plasticity.

   - Prove that if the spike train arrives periodically with a frequency f, the stationary synaptic strength is given by uR with

$$u = \frac{U}{1 + \delta_f (U - 1)} \tag{4}$$

$$R = \frac{1 - \delta_r}{1 + \delta_r (u - 1)} \tag{5}$$

     with $\delta_r = \exp\left(-1/(f \tau_{rec})\right)$, $\delta_f = \exp\left(-1/(f \tau_{facil})\right)$

   - Compare this result to the one of the previous tutorial item Consider a two population network with couplings $G_{EE} = 1$, $G_{EI} = 0.5$, $G_{IE} = 0.5$, $G_{II} = 1$ and external inputs $I_E = 0.2$, $I_I = 0.42$. Use $\tau_{facil} = 100$, $\tau_{rec} = 5$, $U = 0.1$.
     Evaluate the firing rates of the populations. Is the solution unique? What happens if I_E is stronger?