LASCON 2020
Reduced models and Dynamical Systems Analysis
Tutorial Project I

Part I

Consider the following modified *logistic equation with a threshold* that can be used to describe the transition from a resting state (V_{rest}) to and activated state (V_{act}), if they exist

$$\frac{dV}{dt} = F(V), \tag{1}$$

where t represents time, V is the dependent variable (e.g., voltage), and the function $F(V)$ is given by

$$F(V) = -r\,V\,\left(1 - \frac{V}{T}\right)\left(1 - \frac{V}{K}\right) + I_{app}, \tag{2}$$

The parameters r, K, T and I_{app} constants, $r, K, T > 0$ and $T < K$. T and K represent the threshold (V_{th}) and saturation (V_{sat}) V levels for the unbiased case (I_{app}). The parameter I_{app} represents the input given to the system (e.g., applied DC current). Both V_{th} and V_{sat} depend on I_{app}. As mentioned above, for $I_{app} = 0$, $V_{th} = T$ and $V_{sat} = K$. The parameter r represents the rate constant (inverse of the time constant for the unbiased case).

1. Write a code to solve numerically the ODE (1)-(2) (or adapt the template code provided in the course website: *LogisticGrowthThreshold.m*). The simulation output for each set of parameter values must be

   (a) A graph of the solution $V(t)$

   (b) The equilibrium value(s) $V_{eq} = \lim_{t\to\infty} V(t)$

   (c) A graph of F as a function of V.

   - Each simulation should be run long enough (large enough value of t) so that $V(t)$ reaches values close enough to V_{eq}, but not too long so the changes in $V(t)$ are clearly shown.
   - Plot the two graphs as two panels in the same graph.

- The axis should be labeled correctly.
- The fonts should be large enough (suggested: "fontsize" = 24)

2. Consider the following parameter values: $r = 1$, $T = 0.25$, $K = 1$. Perform simulations as described above for $V(0) = 0.01$ and three values of I: $I = 0$, $I = 0.05$, $I = 0.1$.

3. Consider the following parameter values and initial condition: $r = 1$, $T = 0.4$, $K = 1$, $I = [0, 0.2]$ with intervals $\Delta I = 0.02$ (11 values), and $X(0) = 0.25$

   (a) Simulate the model as described above in **ascending** order of the values of I_{app}. Plot the graph of V_{eq} as a function of I.

   (b) Simulate the model as described above in **descending** order of the values of I_{app}. Plot the graph of V_{eq} as a function of I_{app}.

4. Consider the following parameter values and initial condition: $r = 1$, $T = 0.4$, $K = 1$, $I_{app} = [0, 0.2]$ with intervals $\Delta I = 0.02$ (11 values)

   (a) Simulate the model as described above in **ascending** order of the values of I. For $I = 0$ use $V(0) = 0$. For $I_{app} > 0$ set $V(0)$ equal to V_{eq} in the simulation for the previous value of I_{app}.

   (b) Simulate the model as described above in **descending** order of the values of I_{app}. For $I_{app} = 0.2$ set $V(0)$ equal to the value of V_{eq} computed in the previous simulation for $I_{app} = 0.2$. For $I_{app} < 0.2$ set $V(0)$ equal to V_{eq} in the previous simulation.

   (c) Plot a single graph with all the values of V_{eq} as a function of I_{app}.

   (d) Yes, I know Q4 looks very similar to Q3. Yet,...

Part II

Show that the persistent sodium model (3) exhibits histeresis.

$$\frac{dV}{dt} = I_{app} - G_{Nap} p_\infty(V)(V - E_{Na}) \tag{3}$$

where t is time (ms) and V is voltage (mV). You may write your own code or adapt the template code provided in the course website: *Nap.m*. Use the following functions and parameter values

$$p_\infty(V) = \frac{1}{1 + exp(-(V + 38)/6.5)},$$

$C = 1$ $\mu F/\text{cm}^2$, $E_L = -65$ mV, $E_{Na} = 55$ mV and $G_L = 0.25$ mS/cm^2. The parameters G_p (mS/cm^2) and I_{app} (μA/cm^2) are free. It goes without saying, feel free to change the other parameter values as you see fit.

Part III

Compute the bifurcation diagram of the model in Part I.

Part IV

Compute the bifurcation diagram of the model in Part II.