LASCON 2020
Reduced models and Dynamical Systems Analysis
Tutorial Project II

This project refers to the *leaky integrate-and-fire* (LIF) model (code: LeakyIntegrate-Fire.m),

$$C\,\frac{dV}{dt} = -G_L(V - E_L) + Iapp, \tag{1}$$

$$\text{if } V > V_{th}, \text{ then} \qquad V \to V_{reset}$$

the *quadratic integrate-and-fire* (QIF) model (code: QuadraticIntegrateFire.m)

$$C\,\frac{dV}{dt} = -G_{QIF}(V - V_r)(V - V_c) + Iapp, \tag{2}$$

$$\text{if } V > V_{th}, \text{ then} \qquad V \to V_{reset}$$

the *Exponential integrate-and-fire* (ExIF) model (code: ExponentialIntegrateFire.m).

$$C\,\frac{dV}{dt} = -G_{ExIF}(V - V_r) + G_{ExIF}\Delta T e^{(V-V_{rh})/\Delta T} + Iapp, \tag{3}$$

$$\text{if } V > V_{th}, \text{ then} \qquad V \to V_{reset}$$

and their extensions to include *spike-rate adaptation* and additive *white noise* (normally distributed) with mean zero and variance D.

Part I

Determine the excitability properties of the three models by plotting spiking frequency versus I_{app} diagrams. Specifically, determine whether these models exhibit a gradual transition in firing frequency from rest to spiking activity, admitting infinitesimally small frequencies (type I), or, alternatively, they exhibit an abrupt transition (jump) in firing frequency from rest to spiking activity (type II).

Part II

Determine the effects of spike rate adaptation on the firing frequency of the LIF, QIF and ExIF models by plotting graphs of the firing frequency versus τ_{sra}.

The LIF model with spike rate adaptation reads (code: LeakyIntegrateFireSpkRateAdapt.m)

$$C\,\frac{dV}{dt} = -G_L(V - E_L) - g_{sra}(V - E_K) + Iapp, \tag{4}$$

$$\tau_{sra}\frac{dg_{sra}}{dt} = -g_{sra}, \tag{5}$$

$$\text{if } V > V_{th}, \text{ then} \qquad \text{(i) } V \to V_{reset}, \qquad \text{(ii)} g_{sra} \to g_{sra} + \Delta g_{sra}$$

Spike-rate adaptation can be added to the QIF and ExIF models in a similar way. Remember: $\tau = C/G_L = CR$.

Part III

The three codes (LeakyIntegrateFire.m, QuadraticIntegrateFire.m and ExponentialIntegrateFire.m) include an additive white noise term (mean zero and variance D). For $D = 0$, the models and deterministic and spiking activity is created for values of I_{app} large enough for V to cross threshold (spiking-dominated regime). For $D > 0$, white noise can affect existing spiking patterns or create spiking activity in otherwise silent cells (noise-dominated regime).

Determine the ISI (interspike-interval) distributions for the three models for various combinations of I_{app} and D in both regimes.

Part IV

Explain all these results in terms of the phase-space (phase-like) diagrams.