Part I

This part of the project refers to the FitzHugh-Nagumo (FHN) model [1, 2] (code: FitzHughNagumo.m, KSE = 1) using the following modified notation.

\[
\frac{dV}{dt} = -h V^3 + a V^2 - w, \tag{1}
\]

\[
\frac{dw}{dt} = \epsilon [\alpha V - \lambda - w], \tag{2}
\]

where \(h = 3\) and \(a = 2\). For these values, the cubic \(V\)-nullcline has a trough at \(V = 0\) and a peak at \(V = 1\). Other values are possible for \(h\) and \(a\) as long as the \(V\)-nullcline remains cubic. The parameter \(\epsilon > 0\) represents the time scale separation between the two variables. The parameter \(\alpha > 0\) controls the steepness of the \(w\)-nullcline, and can be thought of as capturing the strength of the negative feedback provided by a resonant ionic current, and the parameter \(\lambda\) controls the displacement of the \(w\)-nullcline with respect of the \(V\)-nullcline, and can be thought of as a constant applied current to the \(V\)-equation (why?).

Compute the fixed-points and their stability.

Plot frequency versus \(\lambda\) graphs for \(\alpha = 2\) and \(\alpha = 4\) and \(\epsilon = 0.01\). Determine for which parameter values the Hopf bifurcation controlling excitability is subcritical and supercritical. Can you explain this behavior in terms of the phase-plane diagrams?

What are the effects of increasing \(\epsilon\) (e.g., \(\epsilon = 0.1\) and \(\epsilon = 1\)) on the oscillatory patterns?

What are the effects of adding white noise (normally distributed) with mean zero and variance \(D\) to the \(V\)-equation on the oscillatory patterns? Where in the patterns is the effect more pronounced and why is that? How does the effect of noise vary with \(\epsilon\)?

The code (code: FitzHughNagumo.m) has two extensions of the FHN model (KSE = 2 and KSE = 3). Interpret these models and describe the main differences between them and the standard FHN model.
Part II

This part refers to the 2D version of the Morris-Lecar model [3, 4] described in class (code: MorrisLecar.m and spiketimes.m).

Plot frequency versus $I_{\text{app}}$ graphs for MDL = 1, MDL = 2 and MDL = 3 ($KSE = 1$). What bifurcations are associated with each one and why?

References


