LASCON 2020
Reduced models and Dynamical Systems Analysis
Tutorial Project III

Part I

This part of the project refers to the FitzHugh-Nagumo (FHN) model [1, 2] (code: FitzHughNagumo.m, KSE = 1) using the following modified notation.

$$\frac{dV}{dt} = -h\,V^3 + a\,V^2 - w, \tag{1}$$

$$\frac{dw}{dt} = \epsilon\,[\,\alpha\,V - \lambda - w\,], \tag{2}$$

where $h = 3$ abd $a = 2$. For these values, the cubic V-nullcline has a trough at $V = 0$ and a peak at $V = 1$. Other values are possible for h and a as long as the V-nullcline remains cubic. The parameter $\epsilon > 0$ represents the time scale separation between the two variables. The parameter $\alpha > 0$ controls the steepness of the w-nullcline, and can be thought of as capturing the strength of the negative feedback provided by a resonant ionic current, and the parameter λ controls the displacement of the w-nullcline with respect of the V-nullcline, and can be thought of as a constant applied current to the V-equation (why?).

Compute the fixed-points and their stability.

Plot frequency versus λ graphs for $\alpha = 2$ and $\alpha = 4$ and $\epsilon = 0.01$. Determine for which parameter values the Hopf bifurcation controlling excitability is subcritical and supercritical. Can you explain this behavior in terms of the phase-plane diagrams?

What are the effects of increasing ϵ (e.g., $\epsilon = 0.1$ and $\epsilon = 1$) on the oscillatory patterns?

What are the effects of adding *white noise* (normally distributed) with mean zero and variance D to the V-equation on the oscillatory patterns? Where in the patterns is the effect more pronounced and why is that? How does the effect of noise vary with ϵ?

The code ((code: FitzHughNagumo.m) has two extensions of the FHN model (KSE = 2 and KSE = 3). Interpret these models and describe the main differences between them and the standard FHN model.

Part II

This part refers to the 2D version of the Morris-Lecar model [3, 4] described in class (code: MorrisLecar.m and spiketimes.m).

Plot frequency versus I_{app} graphs for MDL = 1, MDL = 2 and MDL = 3 (KSE = 1). What bifurcations are associated with each one and why?

References

[1] R. FitzHugh. Thresholds and plateaus in the Hodgkin-Huxley nerve equations. *J. Gen. Physiol.*, 43:867–896, 1960.

[2] J. S. Nagumo, S. Arimoto, and S. Yoshizawa. An active pulse transmission line simulating nerve axon. *Proc. IRE*, 50:2061–2070, 1962.

[3] H. Morris, C.and Lecar. Voltage oscillations in the barnacle giant muscle fiber. *Biophysical J.*, 35:193–213, 1981.

[4] J. Rinzel and G. B. Ermentrout. Analysis of neural excitability and oscillations. *In Methods in Neural Modeling. Koch, C. and Segev, I. (Eds.), second edition. MIT Press: Cambridge, Massachusetts*, pages 251–292, 1998.